Int J Thermophys (2010) 31:1789–1794
DOI 10.1007/s10765-010-0835-4

Investigating the Inconsistency of ITS-90 for SPRTs in the Subrange 0 °C to 419.527 °C

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Received: 6 March 2010 / Accepted: 22 September 2010 / Published online: 12 October 2010

Abstract The subrange inconsistency is a significant factor to uncertainty in the standard platinum resistance thermometer (SPRT) subranges of the International Temperature Scale of 1990 (ITS-90). This paper investigated the subrange inconsistency between the water–zinc and water–aluminum subranges. The calibration data for 60 SPRTs from four manufacturers were analyzed, and the result confirms that the coefficient c in the interpolation of ITS-90 is available to determine the subrange inconsistency in this temperature range again. The inconsistency, Δt, can be simply equal to 59.83c.

Keywords Fixed points · Interpolation equation · SPRTs · Subrange inconsistency

1 Introduction

The International Temperature Scale of 1990 (ITS-90) [1] is defined by sets of the defining temperature fixed points, interpolation equations, and interpolation thermometers. ITS-90 provides more subranges available for the calibration of standard platinum resistance thermometers (SPRTs) than the International Temperature Scale of 1968, Amended Edition of 1975. These subranges give more flexibility for the calibration of SPRTs. Meanwhile, the overlap regions of subranges inherently

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generate inconsistencies from the use of different mathematical relations, which are now more formally called Type-I non-uniqueness [2]. For the temperature range of 0 °C to 660.323 °C, two relations cover the subrange of 0 °C to 419.527 °C, yielding a subrange inconsistency between the relations. Crovini [3] investigated eight SPRTs in this subrange, indicating an inconsistency of less than 0.5 mK, as did Strouse [4] for 13 SPRTs. Moiseeva and Pokhodun [5] reported that four investigated Russia-made thermometers of conventional structure had a subrange inconsistency of less than 0.5 mK, and two thermometers of another structure demonstrated a subrange inconsistency greater than 5 mK. White and Strouse [6] present both the numerical results and some observations about subrange inconsistencies, and suggest that a single constraint on the value of ratios might be more useful. Zhiru et al. [7]. developed a relation with which the inconsistency of SPRTs can be easily obtained. Sixty SPRTs examined for their inconsistency indicated good agreement with the predicted results by the relation [7]. In this paper, the developed relation was examined for 60 SPRTs calibrated in the National Institute of Metrology (NIM) in the subrange of 0 °C to 419.527 °C, and confirms the reliability of the relation again.

2 Relation to Describe Inconsistency

ITS-90 defines 11 partially overlapping subranges generating the situation that two different relations are permitted to be used simultaneously for the same subrange. For the subrange 0 °C to 419.527 °C, two interpolation equations are available. One is for the temperature range 0 °C to 660.323 °C:

$$W(t) - W_{r1}(t) = a[W(t) - 1] + b[W(t) - 1]^2 + c[W(t) - 1]^3 \quad (1)$$

where $W(t) = R(t)/R_{tp}$, $R(t)$ is the resistance of a tested SPRT at the measured temperature t, and R_{tp} is the resistance of the same SPRT at the triple point of water. $W_r(t)$ is the reference resistance ratio, and a, b, and c are coefficients. The SPRT is calibrated at the triple point of water, the freezing point of tin, the freezing point of zinc, and the freezing point of aluminum to derive the coefficients a, b, and c.

Another available interpolation equation is for the temperature range 0 °C to 419.527 °C for the same SPRT,

$$W(t) - W_{r2}(t) = a^*[W(t) - 1] + b^*[W(t) - 1]^2 \quad (2)$$

The SPRT is calibrated at the triple point of water, the freezing point of tin, and the freezing point of zinc to obtain the coefficients a^* and b^*.

Subtracting Eq. 2 from Eq. 1 yields

$$W_{r2}(t) - W_{r1}(t) = (a - a^*)[W(t) - 1] + (b - b^*)[W(t) - 1]^2 + c[W(t) - 1]^3 \quad (3)$$

At the triple point of water, the freezing point of tin and the freezing point of zinc, W_{tp}, W_{Sn}, and W_{Zn} are definitely three roots of the following equation:

$$(a - a^*)[W(t) - 1] + (b - b^*)[W(t) - 1]^2 + c[W(t) - 1]^3 = 0 \tag{4}$$

Therefore, the right-hand side of Eq. 3 shall be expressed as

$$W_{r2}(t) - W_{r1}(t) = c[W(t) - 1]\,[W(t) - W_{Sn}]\,[W(t) - W_{Zn}] \tag{5}$$

Let

$$\Delta t = (W_{r2}(t) - W_{r1}(t))/(\mathrm{d}W_r(t)/\mathrm{d}t) \tag{6}$$

Thus, the inconsistency over the 0 °C to 419.527 °C range can be expressed as

$$\Delta t = \{c[W(t) - 1]\,[W(t) - W_{Sn}]\,[W(t) - W_{Zn}]\}/\mathrm{d}W_r/\mathrm{d}t \tag{7}$$

The reference function $W_r(t)$ is very close to $W(t)$ for normal SPRTs. The difference is estimated to be not larger than 10^{-3}. Therefore, Eq. 7 can be approximated by

$$\Delta t \cong \Delta t' = \{c[W_r(t) - 1]\,[W_r(t) - W_{Sn}]\,[W_r(t) - W_{Zn}]\}/(\mathrm{d}W_r(t)/\mathrm{d}t) \tag{8}$$

$\Delta t'$ is zero at W_r equal to W_{tp}, W_{Sn}, and W_{Zn}. As a result, Eq. 8 has two extreme points over the temperature range 0 °C to 419.527 °C [7]. These are:

$$\Delta t'_1 = 59.83c \quad \text{at } 93.15\ ^\circ\text{C} \tag{9}$$

and,

$$\Delta t'_2 = -39.88c \quad \text{at } 337\ ^\circ\text{C} \tag{10}$$

As $|\Delta t'_1| > |\Delta t'_2|$, $\Delta t'_1$ is the maximum value of $\Delta t'$. The details of the uncertainty analysis were given by Zhiru et al. [7].

3 Examination of Inconsistency

Reference [7] reported the examination of Δt and $\Delta t'$ for 65 SPRTs. The examination showed that the differences were not larger than 0.01 mK between Δt and $\Delta t'$ for all the examined SPRTs. In this section, other 60 SPRTs were examined for the inconsistency. Those SPRTs had been calibrated in NIM from 0 °C to 660.323 °C from 2001 to 2007. The SPRTs were produced by three Chinese companies and Hart Scientific (Salt Lake City, USA). Their nominal resistances were 0.25 Ω, 2.5 Ω, and 25 Ω. Table 1 described the sources and resistances of the examined SPRTs. Tables 2 to 5

Table 1 Sources and resistances of the examined thermometers

Company	SPRTs	Distribution of SPRTs		
		0.25 Ω	2.5 Ω	25 Ω
Da Fang	18	6	6	6
Yunnan Instrument	29	11	3	15
Const	4	–	–	4
Hart Scientific	9	1	1	7

Table 2 Inconsistency of the SPRTs from the Yunnan Instrument factory

SPRT No.	Δt (mK)	$\Delta t'$ (mK)
92800	−0.214	−0.214
92803	0.777	0.779
92196	0.097	0.097
92103	−1.584	−1.586
92118	−0.003	−0.004
92827	0.087	0.087
91727	0.195	0.196
91741	0.212	0.213
90722	0.405	0.403
91724	0.436	0.440
92281	0.273	0.272
92226	0.224	0.220
91726	0.246	0.250
00201	−0.123	−0.117
03175	0.471	0.471
92025	0.306	0.306
94825	0.630	0.629
78296	0.556	0.548
18207	0.960	0.960
92026	0.306	0.306
90301	0.352	0.352
94806	0.509	0.510
1311	0.944	0.945
95065	0.131	0.132
03252	0.542	0.543
03259	0.299	0.300
03293	0.673	0.673
03312	0.303	0.299
94804	0.514	0.514

give our calculations of Δt and $\Delta t'$ of SPRTs. Figure 1 is a histogram of the subrange inconsistency frequency distribution. Figure 1 shows the subrange inconsistency of all SPRTs is less than ± 1 mK; and for 78 % of SPRTs, the inconsistency is less than

Table 3 Inconsistency of the SPRTs from Da Fang

SPRT No.	Δt (mK)	$\Delta t'$ (mK)
97130	0.157	0.157
97131	−0.031	−0.031
97129	−0.006	−0.007
97127	0.132	0.132
97115	0.072	0.066
97120	0.185	0.179
97166	0.090	0.087
97179	−0.215	−0.209
97178	0.100	0.101
94844	−0.104	−0.104
95034	−0.241	−0.241
97159	0.565	0.565
97116	−0.052	−0.051
0389	0.559	0.559
97182	0.073	0.073
92165	−0.357	−0.357
92176	0.447	0.448
95055	0.278	0.277

Table 4 Inconsistency of the SPRTs from Const

SPRT No.	Δt (mK)	$\Delta t'$ (mK)
69008	0.009	0.009
69006	−0.267	−0.267
69013	0.116	0.115
69001	−0.088	−0.088

±0.5 mK. Tables 2 to 5 show that the differences between Δt and $\Delta t'$ are not larger than 0.01 mK for all examined SPRTs. Those differences are much smaller than the criterion value of 0.5 mK of the inconsistency. This good agreement confirms further the availability of the simple relation, $\Delta t_1' = 59.83c$, for the inconsistency evaluation of an individual SPRT. This relation provides a convenient and reliable method to check the inconsistency of a calibrated SPRT from 0 °C to 419.527 °C by a simple multiple calculation of the calibration coefficient c of Eq. 1 as 59.83.

4 Conclusion

Sixty SPRTs were examined for their inconsistency in the subrange 0 °C to 419.527 °C by the application of the relation derived by Zhiru et al. [7]. The examined SPRTs were calibrated over the temperature range from 0 °C to 660.323 °C from 2001 to 2007 in NIM. Those SPRTs were produced by four different companies. The examination demonstrated that the calculated results of using the relation, $\Delta t_1' = 59.83c$, had good

Table 5 Inconsistency of the SPRTs from Hart Scientific

SPRT No.	Δt (mK)	$\Delta t'$ (mK)
1558	0.218	0.218
1557	0.132	0.132
1060	0.008	0.009
1418	0.242	0.242
985046	−0.071	−0.071
1386	0.551	0.551
1389	0.332	0.331
1562	−0.398	−0.398
1038	0.064	0.068

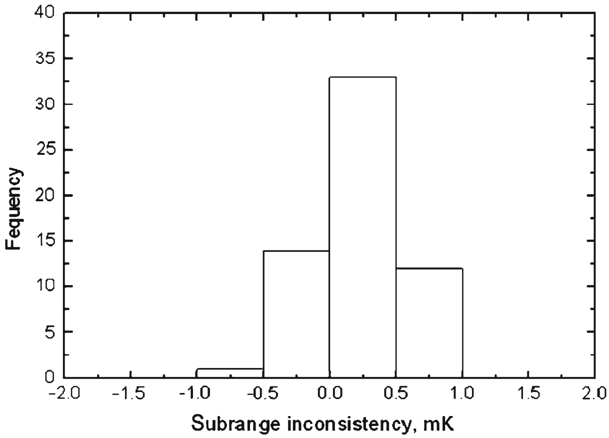

Fig. 1 Histogram of subrange inconsistency frequency distribution

agreement with the subrange inconsistency results calculated directly with the interpolation equations. This good agreement further confirms the reliability of the convenient relation in the evaluation of the subrange inconsistency of SPRTs.

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